GCE

Mathematics A

H230/01: Pure Mathematics and Statistics

Advanced Subsidiary GCE

Mark Scheme for Autumn 2021

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Text Instructions

1. Annotations and abbreviations

Annotation in RM assessor	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	**Meaning**
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value **is given** in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value **is not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for *g* should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	Guidance
1	(a)		$\frac{1}{2} \times 3 \times 4 \times \sin 30$ $= 3$	M1 A1 [2]	
	(b)		$AC^2 = 3^2 + 4^2 - 2\times3\times4\times\cos 30^o$ (= 4.22) $AC = 2.05$ (3 sf)	M1 A1 [2]	or $AC = \sqrt{3^2 + 4^2 - 2\times3\times4\times\cos 30^o}$ Correct expression AC^2 or AC
	(c)		$\frac{\sin C}{3} = \frac{\sin 30}{\text{their} 2.05}$ oe eg $\frac{\sin C}{3} = \frac{\sin 30}{\sqrt{4.22}}$ $ACB = \sin^{-1}(\frac{3\sin 30}{\text{their} 2.05})$ (= $\sin^{-1}0.73\ldots$) $3^2 = \text{'2.05'}^2 + 4^2 - 2\times\text{'2.05'}\times4\times\cos ACB$ $ACB = \cos^{-1}\frac{\text{'2.05'}^2+4^2-3^2}{2\times\text{'2.05'}\times4}$ $ACB = 46.9^o$ or 47.0° or 47.1° (3 sf)	M1 M1 M1 M1 A1ft [3]	Correct sin rule using their (b) Attempt inverse sine of $\frac{3\sin 30}{\text{their(b)}}$. May be implied by answer Correct cos rule using their (b) Attempt inverse cos of $\frac{\text{'2.05'}^2+4^2-3^2}{2\times\text{'2.05'}\times4}$ May be implied by answer Allow 47° Condone premature rounding. FT their (b)
2			$\frac{dn}{dt}$ $= -5000t^{-2}$ At $t = 5$, $\frac{dn}{dt} = \frac{-5000}{25}$ ISW (Rate of change of n is) –200	M1 A1 M1 A1 [4]	Attempt differentiate Substitute $t = 5$ into their $\frac{dn}{dt}$ which must include t^{-2}
3	(a)		$x < 1$ $2 < x < 3$ $\{x: x < 1\} \cup \{x: 2 < x < 3\}$	B1 B1 B1f [3]	Any notation Allow $0.4 < x < 1$ Any notation. Lose only one B1 if ≤ seen instead of < ft their (i) dep at least two ranges; allow ≤ SC: (MR '>') $1 < x < 2$; $x > 3$ B0B1 $\{x: 1 < x < 2\} \cup \{x: x > 3\}$ B1
3	(b)		Attempt reflect in y-axis Through (-3, 0.5), (-2, 0.5), (-1, 0.5), (-0.5, -1.5)	M1 A1 [2]	Approx correct shape, orientation & location Allow ± 2mm SC. All four points plotted ± 2mm: B1

3	**(c)**		Curve intersects x-axis only once	**B1** **[1]**	oe
4	**(a)**		$\begin{pmatrix}15\\-12\end{pmatrix}$ or $15\mathbf{i} - 12\mathbf{j}$	**B1** **B1** **[1]**	B1 for each element. Allow **i**, **j** notation without "squiggles"
4	**(b)**		$6r - s = 0$ $s = 6r$ $0^2 + (9r)^2 = 9$ $81r^2 = 9$ $r = \pm\frac{1}{3}$ $r = \frac{1}{3}$ and $s = 2$ or $r = -\frac{1}{3}$ and $s = -2$	**M1** **M1** **A1** **A1** **A1** **[5]**	 Attempt $\lvert\mathbf{a}\rvert^2 = 9$, or $\lvert\mathbf{a}\rvert = 3$; allow in terms of both r and s Allow just $r = \frac{1}{3}$ Correctly paired
5	**(a)**		Maximum speed of the car or model will show consumption eventually becoming negative or model may not apply for above 80 mph	**B1** **[1]**	or, eg, doesn't drive faster than 80, or speed limit Condone eg "Maximum number of miles car can drive"
5	**(b)**		$\frac{d}{dv}(\frac{12}{5}v - \frac{3}{125}v^2) = 0 \quad (\Rightarrow \frac{12}{5} - \frac{6v}{125} = 0)$ $v = 50$ $\frac{d^2}{dv^2}(\frac{12}{5}v - \frac{3}{125}v^2) = -\frac{6}{125}$ when $v = 50$ or any correct method showing that SP is a maximum Maximum speed is 50 mph	**M1** **A1** **M1** **A1**	Attempt differentiate C & equate to 0 Must be correct Units essential. Dep only on 1st M1

5	(b) ctd		**Alternative method 1** $v = -\frac{b}{2a}\left(= -\frac{\frac{12}{5}}{\frac{2\times(-3)}{125}}\right)$ Attempt complete square $v = 50$ Coefficient of v^2 negative, hence stationary point is a maximum Maximum speed is 50 mph	**M1** **A1** **M1** **A1**	 Units essential
			Alternative method 2 $\frac{12}{5}v - \frac{3}{125}v^2 = 0$ ($v = 0$ or 100) & Correct sketch graph & $v = 50$ $v = 50$ seen on graph as giving maximum Maximum speed is 50 mph	**M1** **B1** **M1** **A1** **[4]**	Working must be seen NB. This mark can be gained without working to justify the graph. Units essential
5	(c)		$v = 0$ does not give $C = 0$ oe	**B1** **[1]**	They will not consume fuel at 0 mph oe
5	(d)		eg $k(\frac{12}{5}v - \frac{3}{125}v^2)$ with any $k > 1$	**B1** **B1**	or "Increase both constants by the same factor" B1B1 or with numerical value of k (> 1) B1B1 SC: "Increase both constants" B1B0
			Alternative method eg $(1 + k)(\frac{12}{5}v - \frac{3}{125}v^2)$ where $k > 0$	**B1** **B1**	
				[2]	

6			$k = \frac{50}{3.6^3}$ or $\frac{3125}{2916}$ or 1.07(167) '1.07'$v^3 = 225$ or $v^3 = \frac{225}{\text{'1.07'}}$ oe or $v = \sqrt[3]{\frac{225}{\text{'1.07167'}}}$	**M1** **M1**	Attempt find k. Must involve division, and cube or cube root Can be implied by 1.07 seen, or correct v or $v = \sqrt[3]{\frac{225}{\text{their } k}}$ oe SC $k = \frac{50}{3.6^2}$ oe Max M1M0A0
			Alternative Method $\frac{v^3}{3.6^3} = \frac{225}{50}$ $\frac{v}{3.6} = \sqrt[3]{\frac{225}{50}}$ or $v = 3.6 \times \sqrt[3]{\frac{225}{50}}$	**M1** **M1**	Attempt use proportion. Must involve cube or cube root Correct expression for v
			$v = 5.94$ (3 sf) (Wind speed = 5.94 m/s)	**A1** **[3]**	Allow without units
7	**(a)**		Plot log P against log Q	**B1** **[1]**	Either way round
7	**(b)**		log Q = log a + n log P So plotting log P against log Q gives a straight line $Y = m\,X + c$ with gradient n Y = log Q and X = log P	 **M1** **A1** **[2]**	(Draw a line of best fit) Comparison with $Y = mX + c$ Clarify which way round
8	**(a)**		$2^3 < 3^2$	**B1** **[1]**	or eg $2^3 \geq 3^2$, $8 \geq 9$ Not true. Allow if poorly expressed.
8	**(b)**		$n(n + 2) + 1$ $(= n^2 + 2n + 1)$ $= (n + 1)^2$ But n is odd so $n + 1$ is even, so this is (even number)^2	**M1** **A1** **M1** **A1**	

8	**(b) ctd**		**Alternative Method 1** Let $m = 2k - 1$ and $n = 2k + 1$ where k is a positive integer $(2k - 1)(2k + 1) + 1$ $(= 4k^2)$ $= (2k)^2$ or $2^2 \times k^2$ This is (even number)^2	 **M1** **M1** **A1** **A1**	 Allow $2n - 1$ and $2n + 1$ Condone omission of "where k is a positive integer"
			Alternative Method 2 Let $m = 2k + 1$ and $n = 2k + 3$ where k is a positive integer $(2k + 1)(2k + 3) + 1$ $(= 4k^2+8k+4 = 4(k^2 + 2k + 1))$ $= [2(k + 1)]^2$ or $2^2 \times (k + 1)^2$ This is (even number)^2	 **M1** **M1** **A1** **A1**	 Allow $2n + 1$ and $2n + 3$ Condone omission of "where k is a positive integer"
				[4]	
9			$y = mx + 2$ $mx + 2 = x^2 - x + 6$ $x^2 - (m + 1)x + 4 = 0$ Equal roots, hence $(m + 1)^2 - 16 = 0$ $m = -5$ inadmissible $m = 3$ Equation is $y = 3x + 2$	**M1** **M1** **A1** **M1** **B1** **A1**	seen or implied Attempt solve eqns of line and curve simultaneously Attempt use $b^2 - 4ac = 0$
			Alternative Method grad of tangent $= 2x - 1$ Where tgt touches curve $y = (2x-1)x + 2$ Hence $(2x - 1)x + 2 = x^2 - x + 6$ $x^2 = 4$ $x = -2$ gives gradient $= -5$, so reject $x = 2$ gives gradient $= 3$ Equation is $y = 3x + 2$	**M1** **M1** **M1** **A1** **B1** **A1**	Must be correct Allow M1 for just $y = (2x-1)x + 2$ oe
				[6]	

Question			Answer	Marks	Guidance
10	(a)		(55) 115 (60) | (230) (65) (35) 70 | 170 120 (150) 130 | (400)	B1 B1 [1]	B1 for 10 correct numbers
10	(b)	(i)	$\frac{55}{400}$ ISW or $\frac{11}{80}$ or 0.1375 or 0.138 (3 sf)	B1 [1]	NB If figures for A in the table are correct, but those for B and C are wrong, correct answers to parts (b)(i), (ii) can be obtained correctly.
10	(b)	(ii)	$\frac{55}{120}$ ISW or $\frac{11}{24}$ or 0.458 (3 sf)	B1 [1]	
10	(c)	(i)	Kareem because, eg the 2nd digit of each no. is the same as the 1st digit of the next	B1 [1]	oe. Allow Kareem because the numbers are not independent.
10	(c)	(ii)	Only 820 students or 850 > 820	B1 [1]	oe
11	(a)		Those in age group 8 to 9 in 2001 will be in age group 18 to 19 in 2011	B1 [1]	or, eg, "Same people" May be implied
11	(b)	(i)	Few people joined (or left) this group oe or "Not much change in this group" or “Most have lived there since 2001”	B1 [1]	LAs with a high number of 8-9s in 2001 have a high number of 18-19s in 2011 Most have stayed in the same area since childhood
11	(b)	(ii)	(No) The nos of 18-19's in 2011 is (largely) caused by the nos of 8-9s in 2001, (but this cannot be inferred from the correlation).	B1 [1]	(No) There is a link. They are the same people Must state or imply that there is a link between x and y in this context.
11	(c)	(i)	3 (or 4 or 5) of the highest five y-values	B1 [1]	Must not include right-most point

Question			Answer	Marks	Guidance
11	(c)	(ii)	In these LAs, the no. of 18-19s in 2011 is more than the no. of 8-9s in 2001 In these LAs there are large numbers of 18+	B1 [1]	eg, the no. of people in this group in 2011 is more than it was in 2001 Students are aged 18+ so they cause an increase in this age group Allow numerical example. May be implied Not just "These are outliers" oe
11	(d)		Right most point indicated	B1 [1]	
12	(a)		$n = 50$; $a = \frac{5}{6}, b = \frac{1}{6}$; or $a = \frac{1}{6}, b = \frac{5}{6}$,	B1 [1]	or $\left(\frac{5}{6}+\frac{1}{6}\right)^{50}$ or $\left(\frac{1}{6}+\frac{5}{6}\right)^{50}$
12	(b)		$H_0: p = \frac{1}{6}$ allow 0.167 or 0.17 $H_1: p < \frac{1}{6}$ p = P(dice shows a 2 (on any throw) or p = proportion of 2s thrown	B1 B1 [2]	One error, eg undefined p or 2-tail or $H_1: p > \frac{1}{6}$: B1B0 Two errors, eg $H_1: p > \frac{1}{6}$ and undefined p: B0B0 Cannot be scored in part (c)
12	(c)		B(50, $\frac{1}{6}$) Attempt P($X \leq a$) for $1 \leq a \leq 20$ P($X \leq 4$) = 0.0643 P($X \leq 3$) = 0.0238 Rejection region is ≤ 3 twos or < 4 twos	B1 M1 A1 A1	stated or implied or P($X < 5$) = 0.0643 or P($X < 4$) = 0.0238 Must see both probabilities correct oe. Allow $X \leq 3$ or $X < 4$
			Alternative method ft if 12(b) $H_1: p > \frac{1}{6}$ B(50, $\frac{1}{6}$) Attempt P($X \geq a$) for $1 \leq a \leq 20$ P($X \geq 14$) = 0.0307 P($X \geq 13$) = 0.0627 Rejection region is ≥ 14 twos or > 13 twos	 B1 M1 A1 A1 [4]	 stated or implied Must see both probabilities correct oe. Allow $X \geq 14$ or $X > 13$

13	**(a)**		$\frac{1}{12} + \frac{1}{4} + p + 3p = 1$ $p = \frac{1}{6}$ (2,3) (3,2) (3,3) $\frac{1}{6} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{6} + \frac{1}{2} \times \frac{1}{2}$ $\frac{5}{12}$	**M1** **A1** **M1** **M1** **A1** **[5]**	soi. Allow M1 for any two correct pairs and no incorrect pairs or for all three correct pairs and at most one incorrect pair All correct and added, ft their p.
13	**(b)**		$\frac{1}{2}(1 - 0.02)$ $= 0.49$	**M1** **A1** **[2]**	

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